

1 A Strongly Polynomial Algorithm for Linear 2 Programs with at most Two Non-zero Entries per 3 Row or Column

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14 — Abstract —

15 We give a strongly polynomial algorithm for minimum cost generalized flow, and hence for optimizing
16 any linear program with at most two non-zero entries per row, or at most two non-zero entries
17 per column. Primal and dual feasibility were shown by Végh (MOR '17) and Megiddo (SICOMP
18 '83) respectively. Our result can be viewed as progress towards understanding whether all linear
19 programs can be solved in strongly polynomial time, also referred to as Smale's 9th problem.

20 Our approach is based on the recent primal-dual interior point method (IPM) due to Allamigeon,
21 Dadush, Loho, Natura and Végh (FOCS '22). The number of iterations needed by the IPM is
22 bounded, up to a polynomial factor in the number of inequalities, by the *straight line complexity* of
23 the central path. Roughly speaking, this is the minimum number of pieces of any piecewise linear
24 curve that multiplicatively approximates the central path.

25 As our main contribution, we show that the straight line complexity of any minimum cost
26 generalized flow instance is polynomial in the number of arcs and vertices. By applying a reduction
27 of Hochbaum (ORL '04), the same bound applies to any linear program with at most two non-zeros
28 per column or per row.

29 To be able to run the IPM, one requires a suitable initial point. For this purpose, we develop a
30 novel multistage approach, where each stage can be solved in strongly polynomial time given the
31 result of the previous stage. Beyond this, substantial work is needed to ensure that the bit complexity
32 of each iterate remains bounded during the execution of the algorithm. For this purpose, we show
33 that one can maintain a representation of the iterates as a low complexity convex combination of
34 vertices and extreme rays. Our approach is black-box and can be applied to any log-barrier path
35 following method.

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